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Sub-band Coding of Digital Images Using Symmetric Short Kernel Filters and Arithmetic Coding Techniques

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ABSTRACT

Development of a new and efficient sub-band coding of digital images is reported. This includes 1) the design of short kernel symmetric analysis/synthesis filter banks with perfect reconstruction capability; and 2) entropy coding of the sub-bands by using arithmetic coding in combination with DPCM coding of the lowest band and PCM coding of the higher bands.

1. INTRODUCTION

Subband coding [1], in general, refers to the class of coding techniques where by parallel application of a set of filters (i.e., analysis filters) the input signal is decomposed into several narrowbands. The resulting bands are then decimated and coded separately for the purpose of transmission. For reconstruction, decimated signals are decoded, interpolated, and filtered (i.e.; synthesis filtering) before being added to reproduce the original signal. This approach, in general, demands the design of sophisticated band-pass filters to minimize the perception of aliasing effects. In a much simpler approach, Esteban et al. [2] introduced the concept of Quadrature Mirror Filters (QMFs) which, in the absence of channel and quantization noise, permits an alias-free and near perfect reconstruction of the input signal. Theoretical extension of one-dimensional (1-D) QMF filtering to multi-dimensional cases has been treated by Vetterli [3]. Moreover, application of two-dimensional (2-D) QMF concept to the sub-band coding of digital images has recently received considerable attention and it has been shown to be highly effective for image compression [4]-[8]

A disadvantage with the classical QMF approach, as described in [2], is that it does not permit reconstruction to be exact, although, amplitude distortion can be made very small by using long tap filters. For video and digital image applications use of such long tap filters, while not providing any significant coding gain, may increase the hardware complexity.

In this paper, analysis/synthesis filter banks are realized by using symmetric short kernel filters with perfect reconstruction capability. Furthermore, an adaptive sub-band coding scheme which uses arithmetic coding in combination with the DPCM coding of the lowest band and PCM coding of upper band signals is also presented. In Section II principle of filtering operation is discussed. Our proposed coding scheme is then described in Section 3. Simulation results are covered in Section 4. Summary and conclusions appear in Section 5.

2. Filtering: Exact Reconstruction Filter banks

Figure 1 depicts the principle elements of a two-channel analysis/synthesis filter bank system. As shown, the input signal $x(n)$, is filtered into two bands, which are then decimated by a factor of 2. At the synthesis side, the signals $y_0(n)$ and $y_1(n)$ are interpolated (i.e.; by inserting zeros between successive samples), filtered and added together to reproduce the original signal. Output $Y(z)$ in this figure can be written as:

$$Y(z) = G_0(z)Y_0(z) + G_1(z)Y_1(z) \tag{2.1}$$

where

$$Y_0(z) = \frac{1}{2}[H_0(z)X(z) + H_0(-z)X(-z)] \tag{2.2}$$

$$Y_1(z) = \frac{1}{2}[H_1(z)X(z) + H_1(-z)X(-z)] \tag{2.3}$$

Substitution of Eqs. (2.2)-(2.3) into (2.1) yields

$$Y(z) = 1/2\,[G_0(z)H_0(z) + G_1(z)H_1(z)]\,X(z)$$

$$+\ 1/2\,[G_0(z)H_0(-z) + G_1(z)H_1(-z)]\,X(-z) \tag{2.4}$$

From Eq. (2.4) it is easily seen that aliasing is removed if the synthesis filters are defined as

$$G_0(z) = H_1(-z) \tag{2.5.1}$$

$$G_1(z) = -H_0(-z) \tag{2.5.2}$$

For this class of synthesis filters, perfect reconstruction of the input signal requires that

$$\Delta(z) = P(z) - P(-z) = 2z^{-m} \tag{2.6}$$

where

$$P(z) = H_0(z)H_1(-z). \tag{2.7}$$

is called the product filter, and m is the delay introduced by the combination of analysis/synthesis filter banks.

2.1 Design of Short Kernel Filter Bank

The classical approach to the design of analysis/synthesis filter banks which removes aliasing and approximates Eq. (2.6) is based on the Quadrature Mirror Filter (QMF) concept [2]. This approach makes use of even, symmetrical FIR filters. In addition, QMF's are defined to be mirror image of one another i.e.; $H_1(z) = H_0(-z)$. For this class

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of analysis/synthesis filter banks it is not possible to obtain exact reconstruction if FIR filters greater than length two are used. As an alternative, Smith and Barnwell [9] have proposed a design technique which consists of designing the product filter P(z), and then factoring the result into separate analysis synthesis filter banks (e.g.; minimum and maximum phase components). This technique, in principle, does not lead to linear phase analysis/synthesis filters, and it may not be suitable for image coding applications where the use of symmetric filters is more desirable.

In this paper we address the design of symmetric short tap filters. Our design procedure is based on the factorization of product filter $P(z)$ into linear phase components as suggested by Vetterli [10]. Referring to Eqs. (2.6)-(2.7), a number of observations can be made. First, from Eq. (2.7) note that if both $H_0(z)$ and $H_1(-z)$ are linear phase filters, then their product P(z) also becomes a linear phase filter. Second, Eq. (2.6) can be interpreted as a constraint on the coefficients of P(z). It then follows that product filters with an odd number of coefficients can only satisfy (2.6).

Our design strategy is to choose a half-band product filter P(z) as shown below

$$P(z) = a_0 + a_2 z^{-2} + \ldots + a_{2p-2} z^{-2p+2} + z^{-2p+1}$$

$$+ a_{2p-2} z^{-2p+2} + \ldots\ldots + a_0 z^{-4p+2} \quad (3.1)$$

and optimize the coefficients based on a series of constraints on the location of the zeros. Note that for a P(z) of length 4p-1, there are only p coefficients to be optimized and the result will yield two filters of length 2p. Among the design constraints one can impose is to have filters with integer power of two coefficients.

Case p = 1;

This is the simplest filter bank and the form of P(z) is

$$P(z) = a_0 + z^{-1} + a_0 z^{-2} \quad (3.2)$$

The only solution that is a low pass filter is:

$$P(z) = (1/2 + 1/2 z^{-1})\,(1/2 + 1/2 z^{-1}). \quad (3.3)$$

This is a unique linear phase filter bank of length two and it is the only QMF solution with exact reconstruction as well (see [9]). Such filters can only be useful if coding accuracy is high, otherwise the type of artifacts introduced by quantization noise makes the resulting picture visually unpleasant.

Case p = 2;

In this case the form of P(z) is

$$P(z) = a_0 + a_2 z^{-2} + z^{-3} + a_2 z^{-4} + a_0 z^{-6} \quad (3.4)$$

One degree of freedom can be easily suppressed by imposing a (double) zero at z = -1. P(z) can then be factorized as

$$P(z) = (1 + z^{-1})(1 + z^{-1})(a_0 - 2a_0 z^{-1}$$

$$+ (1/2 + 2a_0) z^{-2} - 2a_0 z^{-3} + a_0 z^{-4}). \quad (3.5)$$

Note that useful filters exist only for values of a_0 in the interval [-1/8, 0]. For example, by setting $a_0 = -1/16$ we get an additional double zero at -1. In this case, the expression for P(z) becomes

$$P(z) = 1/16\,(1 + z^{-1})^3\,(-1 + 3z^{-1} + 3z^{-2} - z^{-3}) \quad (3.6)$$

This factorization yields two pairs of filters for exact reconstruction

Solution 1 (equal length):

$$H_0(z) = 1/4\,(1 + 3z^{-1} + 3z^{-2} + z^{-3}) \quad (3.7.1)$$

$$H_1(-z) = 1/4\,(-1 + 3z^{-1} + 3z^{-2} - z^{-3}) \quad (3.7.2)$$

$$H_0(z) = 1/4\,(-1 + 3z^{-1} + 3z^{-2} - z^{-3}) \quad (3.7.3)$$

$$H_1(-z) = 1/4\,(1 + 3z^{-1} + 3z^{-2} + z^{-3}) \quad (3.7.4)$$

Solution 2 (unequal length):

$$H_0(z) = 1/8\,(-1 + 2z^{-1} + 6z^{-2} + 2z^{-3} - z^{-4}) \quad (3.8.1)$$

$$H_1(-z) = 1/2\,(1 + 2z^{-1} + z^{-2}) \quad (3.8.2)$$

By comparing the filters in solution 1 and solution 2, it appears that the interpolation process to reconstruct the baseband signal in method 2 is what is commonly known as "linear interpolation". This interpolation scheme is known to produce visually pleasant and smooth outputs. From the point of view of computational complexity both filter banks can be implemented by a limited number of shifts and adds, and thus are computationally more efficient than their traditional QMF counterparts.

3. CODING

For channel coding of sub-bands, the basic approach of DPCM coding of lowest band signal, PCM coding of upper band signals, and run-length coding of the address of non-zero PCM values has been reported by [Gharavi, and Tabatabai 1986]. In order to achieve a higher compression rate, we have extended the above method by using arithmetic coding [11] in combination with DPCM coding of lower band and PCM coding of higher band signals. Arithmetic coding method used in our simulations is referred to as "Q-Coder" and has been described in detail by [Mitchell, and Pennebaker 1987]. The Q-coder has several important properties; it avoids multiplication, and it does not require any predetermined symbol probabilities. The principle characteristics of our coding method is described in the following section.

3.1 Coding of the Lowest Band Signal

Similar to the approach taken in [13], we have combined adaptive DPCM and arithmetic coding. Briefly, in this technique, a Markov-like state is determined from a measure of local image activity, and the Q-coder is used to encode the magnitude of the difference data by using probabilities conditioned on the state. Separately, conditional entropy coding is also applied to encode the sign of error data. Here, for our purposes, we have used a different set of quantization levels and threshold values. This is because the quality of the reconstructed picture depends, to large extent, on the fidelity with which the lowest band signal is coded. Tables I and II, shown below, illustrate the quantizer levels and their corresponding threshold values.

In Table I, for example, all differences from 8 to 10 are mapped to 9.

3.2 Coding of the Higher Bands

Our basic strategy to encode higher band signals is based on using a symmetric PCM quantizer with the following characteristics [5]: 1) it contains a dead-zone, d; 2) an active range where signals falling in this range are uniformly coded into L levels; and 3) a saturation value Y, for signals whose magnitude are above a given threshold t.

By using the above quantizer a binary sequence suitable for the input to the Q-coder is generated. That is, first depending on whether the magnitude of the signal is below or above "d" a "0" or "1" is sent. Note that such a binary sequence, in general, will correspond to the contour information of the higher band signals. For cases when a "1" is sent a code word for the sign and magnitude of the non-zero PCM

TABLES I-II: QUANTIZATION AND THRESHOLD VALUES

Table I

Threshold	Level
5	3
8	6
11	9
15	12
20	17
26	22
34	29
44	38
57	49
74	64
95	83
121	106
175	135
256	255

Table II

Threshold	Level
11	6
21	15
31	25
41	35
52	45
72	60
95	83
180	135
256	255

value is also constructed and transmitted. It consists of a sign bit followed by a "0" for each quantization level skipped, and terminated by a "1" at the desired quantization level.

4. SIMULATION RESULTS

Computer simulations were carried out on two pictures "Lena" and "Pepper" (see Figs. (2)-(3)). The input pictures were first divided into four bands by applying 1-D filters (see Eqs. 3.7.3-3.7.4) along their respective rows and columns. For the lowest band an adaptive DPCM coding in combination with arithmetic coding was used (see Section 3.1). For the higher bands, non-zero PCM values together with their positional information were transmitted by using arithmetic coding (see Section 3.2). The PCM quantizer parameters used in this case were selected as follows: d=8, t=36, Y=42, and L=8.

The results of our proposed coding scheme both in terms of subjective quality of the reconstructed pictures as well as average bit rate and SNR are shown in Table III. (Also see Figs. 4 and 5).

Table III - Bit Rate and SNR for LENA and PEPPERS

Picture	Bit Rate	SNR
LENA	.7	34.56
PEPPERS	.77	33.83

5. SUMMARY AND CONCLUSIONS

In this paper a simple, and efficient method of sub-band coding of digital images has been reported. First, a technique for designing symmetric short tap filters has been presented, and it has been shown that such filters can be easily implemented by using simple arithmetic operations (e.g., addition and multiplication). By applying the above filters the input image has been decomposed into four bands, which have then been coded by using arithmetic coding in combination with DPCM coding of the lowest band and PCM coding of higher bands.

Our simulation results have demonstrated that by using the method mentioned above good quality pictures can be obtained in the range of 0.7 to 0.8 bits/pel.

ACKNOWLEDGEMENT

The authors wish to thank Mike Mills for his help in the preparation of this manuscript.

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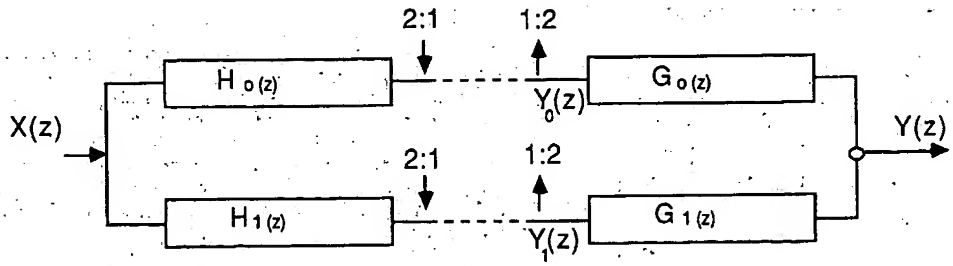

Fig. 1: A Two-Band Analysis/Synthesis Filter Bank System

Fig. 2: Original Lena (512×512)

Fig. 4: Reconstructed Lena 0.7 Bits/Pel

Fig. 3: Original Pepper (512×512)

Fig. 5: Reconstructed Pepper 0.77 Bits/Pel